Mathematica 11.3 Integration Test Results

Test results for the 145 problems in "1.2.2.6 P(x) (d x)^m (a+b x^2+c x^4)^p.m"

Problem 40: Result is not expressed in closed-form.

$$\int \frac{(d\,x)^m \left(A+B\,x+C\,x^2\right)}{a+b\,x^2+c\,x^4}\,dx$$

Optimal (type 5, 368 leaves, 8 steps):

$$\left(\left(C+\frac{2\,A\,c-b\,C}{\sqrt{b^2-4\,a\,c}}\right)(d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\left(b-\sqrt{b^2-4\,a\,c}\right)d\,(1+m)\right)+\left(\left(C-\frac{2\,A\,c-b\,C}{\sqrt{b^2-4\,a\,c}}\right)(d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/\left(\left(b+\sqrt{b^2-4\,a\,c}\right)d\,(1+m)\right)+\frac{2\,B\,c\,(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]}{\sqrt{b^2-4\,a\,c}\left(b-\sqrt{b^2-4\,a\,c}\right)d^2\,(2+m)}-\frac{2\,B\,c\,(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]}{\sqrt{b^2-4\,a\,c}\left(b+\sqrt{b^2-4\,a\,c}\right)d^2\,(2+m)}$$

Result (type 7, 438 leaves):

$$\frac{1}{2 m (1+m) (2+m)} (d x)^m \left(A \left(2+3 m+m^2\right) \text{RootSum}\left[a+b \#1^2+c \#1^4\&, \frac{\text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m}}{b \#1+2 c \#1^3}\&\right]+B (2+m) \text{RootSum}\left[a+b \#1^2+c \#1^4\&, \frac{1}{b \#1+2 c \#1^3}\left(m x+\text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1+m\, \text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1\right)\&\right]+C\, \text{RootSum}\left[a+b \#1^2+c \#1^4\&, \frac{1}{b \#1+2 c \#1^3}\left(m x^2+m^2 x^2+2 m x \#1+m^2 x \#1+2\, \text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1^2+3 m\, \text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1^2+m^2\, \text{Hypergeometric2F1}\left[-m,-m,1-m,-\frac{\#1}{x-\#1}\right] \left(\frac{x}{x-\#1}\right)^{-m} \#1^2+m \left(\frac{x}{\#1}\right)^{-m} \#1^2\right)\&\right]\right)$$

Problem 41: Result unnecessarily involves higher level functions.

$$\int \frac{(d x)^m \left(A+B x+C x^2\right)}{\left(a+b x^2+c x^4\right)^2} \, dx$$

Optimal (type 5, 685 leaves, 10 steps) :

$$\frac{B\,(d\,x)^{2+m}\,\left(b^2-2\,a\,c+b\,c\,x^2\right)}{2\,a\,\left(b^2-4\,a\,c\right)\,d^2\,\left(a+b\,x^2+c\,x^4\right)}+\frac{(d\,x)^{1+m}\,\left(A\,\left(b^2-2\,a\,c\right)-a\,b\,C+c\,\left(A\,b-2\,a\,C\right)\,x^2\right)}{2\,a\,\left(b^2-4\,a\,c\right)\,d\,\left(a+b\,x^2+c\,x^4\right)}+$$

$$\left(c\,\left(2\,a\,C\,\left(2\,b-\sqrt{b^2-4\,a\,c}\,\,(1-m)\right)+A\,\left(b^2\,(1-m)+b\,\sqrt{b^2-4\,a\,c}\,\,(1-m)-4\,a\,c\,(3-m)\right)\right)\right.$$
$$\left.(d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}\,\right)\,d\,(1+m)\right)-$$

$$\left(c\,\left(2\,a\,C\,\left(2\,b+\sqrt{b^2-4\,a\,c}\,\,(1-m)\right)+A\,\left(b^2\,(1-m)-b\,\sqrt{b^2-4\,a\,c}\,\,(1-m)-4\,a\,c\,(3-m)\right)\right)\right.$$
$$\left.(d\,x)^{1+m}\,\text{Hypergeometric2F1}\left[1,\frac{1+m}{2},\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,d\,(1+m)\right)-\left(B\,c\,\left(4\,a\,c\,(2-m)+b\,\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,m\right)\right.$$
$$\left.(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2\,c\,x^2}{b-\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b-\sqrt{b^2-4\,a\,c}\,\right)\,d^2\,(2+m)\right)+\left(B\,c\,\left(4\,a\,c\,(2-m)+b\,\left(b-\sqrt{b^2-4\,a\,c}\,\right)\,m\right)\right.$$
$$\left.(d\,x)^{2+m}\,\text{Hypergeometric2F1}\left[1,\frac{2+m}{2},\frac{4+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}}\right]\right)\Big/$$
$$\left(2\,a\,\left(b^2-4\,a\,c\right)^{3/2}\,\left(b+\sqrt{b^2-4\,a\,c}\,\right)\,d^2\,(2+m)\right)$$

Result (type 6, 999 leaves) :

$$\frac{1}{4\,c\,\left(a+b\,x^2+c\,x^4\right)^3}\,a\,x\,(d\,x)^m\,\left(b-\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)\,\left(b+\sqrt{b^2-4\,a\,c}+2\,c\,x^2\right)$$

$$\Bigg(\Bigg(A\,(3+m)\,\text{AppellF1}\Big[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg((1+m)\,\Bigg(a\,(3+m)\,\text{AppellF1}\Big[\frac{1+m}{2},2,2,\frac{3+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$2\,x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\Big[\frac{3+m}{2},2,3,\frac{5+m}{2},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)$$

$$\text{AppellF1}\Big[\frac{3+m}{2},3,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)+$$

$$x\,\Bigg(\Bigg(B\,(4+m)\,\text{AppellF1}\Big[\frac{2+m}{2},2,2,\frac{4+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg((2+m)\,\Bigg(a\,(4+m)\,\text{AppellF1}\Big[\frac{2+m}{2},2,2,\frac{4+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$2\,x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\Big[\frac{4+m}{2},2,3,\frac{6+m}{2},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)$$

$$\text{AppellF1}\Big[\frac{4+m}{2},3,2,\frac{6+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)+$$

$$\Bigg(C\,(5+m)\,x\,\text{AppellF1}\Big[\frac{3+m}{2},2,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Big/$$

$$\Bigg((3+m)\,\Bigg(a\,(5+m)\,\text{AppellF1}\Big[\frac{3+m}{2},2,2,\frac{5+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]-$$

$$2\,x^2\,\Bigg(\left(b+\sqrt{b^2-4\,a\,c}\right)\,\text{AppellF1}\Big[\frac{5+m}{2},2,3,\frac{7+m}{2},$$

$$-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]+\left(b-\sqrt{b^2-4\,a\,c}\right)$$

$$\text{AppellF1}\Big[\frac{5+m}{2},3,2,\frac{7+m}{2},-\frac{2\,c\,x^2}{b+\sqrt{b^2-4\,a\,c}},\frac{2\,c\,x^2}{-b+\sqrt{b^2-4\,a\,c}}\Big]\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)$$

Problem 105: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{x\,\left(3+2\,x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 66 leaves, 8 steps) :

$$\frac{25\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}+\frac{89\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{72\sqrt{2}}+\frac{4\,\text{Log}[x]}{9}-\frac{1}{9}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 93 leaves):

$$\frac{1}{288}\left(-\frac{300\left(-1+x^2\right)}{3+2x^2+x^4}+128\,\text{Log}[x]-\sqrt{2}\left(89\,i+16\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]+\sqrt{2}\left(89\,i-16\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 106: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{x^3\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 71 leaves, 8 steps):

$$-\frac{2}{9x^2}-\frac{25\left(5+x^2\right)}{72\left(3+2x^2+x^4\right)}-\frac{71\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{216\sqrt{2}}-\frac{13\,\text{Log}[x]}{27}+\frac{13}{108}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 97 leaves):

$$\frac{1}{864}\left(-\frac{192}{x^2}-\frac{300\left(5+x^2\right)}{3+2x^2+x^4}-416\,\text{Log}[x]+\sqrt{2}\left(71\,i+52\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]+\sqrt{2}\left(-71\,i+52\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 107: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{x^5\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 80 leaves, 8 steps):

$$-\frac{1}{9x^4}+\frac{13}{54x^2}+\frac{25\left(7+5x^2\right)}{216\left(3+2x^2+x^4\right)}+\frac{125\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{216\sqrt{2}}+\frac{13\,\text{Log}[x]}{27}-\frac{13}{108}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 105 leaves):

$$\frac{1}{864}\left(-\frac{96}{x^4}+\frac{208}{x^2}+\frac{100\left(7+5x^2\right)}{3+2x^2+x^4}+416\,\text{Log}[x]-\sqrt{2}\left(125\,i+52\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]+\sqrt{2}\left(125\,i-52\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 108: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3x^4+5x^6}{x^7\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 87 leaves, 8 steps):

$$-\frac{2}{27x^6}+\frac{13}{108x^4}-\frac{13}{54x^2}+\frac{25\left(1-7x^2\right)}{648\left(3+2x^2+x^4\right)}-\frac{1237\,\text{ArcTan}\left[\frac{1+x^2}{\sqrt{2}}\right]}{1944\sqrt{2}}+\frac{61\,\text{Log}[x]}{243}-\frac{61}{972}\text{Log}\left[3+2x^2+x^4\right]$$

Result (type 3, 110 leaves):

$$\frac{1}{7776}\left(-\frac{576}{x^6}+\frac{936}{x^4}-\frac{1872}{x^2}-\frac{300\left(-1+7x^2\right)}{3+2x^2+x^4}+1952\,\text{Log}[x]+\sqrt{2}\left(1237\,i-244\sqrt{2}\right)\text{Log}\left[1-i\sqrt{2}+x^2\right]-\sqrt{2}\left(1237\,i+244\sqrt{2}\right)\text{Log}\left[1+i\sqrt{2}+x^2\right]\right)$$

Problem 109: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 248 leaves, 12 steps):

$$38x+\frac{19x^3}{3}-\frac{17x^5}{5}+\frac{5x^7}{7}+\frac{25x\left(3+5x^2\right)}{8\left(3+2x^2+x^4\right)}+$$

$$\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{16}\sqrt{\frac{1}{2}\left(262771+618291\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]+$$

$$\frac{1}{32}\sqrt{\frac{1}{2}\left(-262771+618291\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 145 leaves):

$$38 x+\frac{19 x^3}{3}-\frac{17 x^5}{5}+\frac{5 x^7}{7}+\frac{25 x \left(3+5 x^2\right)}{8 \left(3+2 x^2+x^4\right)}-\frac{\left(352 i+1339 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{16 \sqrt{2-2 i \sqrt{2}}}-\frac{\left(-352 i+1339 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{16 \sqrt{2+2 i \sqrt{2}}}$$

Problem 110: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^6 \left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 237 leaves, 12 steps):

$$19 x-\frac{17 x^3}{3}+x^5+\frac{25 x \left(3-x^2\right)}{8 \left(3+2 x^2+x^4\right)}+\frac{3}{16} \sqrt{\frac{3}{2} \left(-8669+5011 \sqrt{3}\right)} \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]-\frac{3}{16} \sqrt{\frac{3}{2} \left(-8669+5011 \sqrt{3}\right)} \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]+\frac{3}{32} \sqrt{\frac{3}{2} \left(8669+5011 \sqrt{3}\right)} \text{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)} x+x^2\right]-\frac{3}{32} \sqrt{\frac{3}{2} \left(8669+5011 \sqrt{3}\right)} \text{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)} x+x^2\right]$$

Result (type 3, 132 leaves):

$$19 x-\frac{17 x^3}{3}+x^5-\frac{25 x \left(-3+x^2\right)}{8 \left(3+2 x^2+x^4\right)}+\frac{9 \left(90 i+31 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{16 \sqrt{2-2 i \sqrt{2}}}+\frac{9 \left(-90 i+31 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{16 \sqrt{2+2 i \sqrt{2}}}$$

Problem 111: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^4 \left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 232 leaves, 12 steps):

$$-17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}-\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{16}\sqrt{\frac{1}{2}\left(14395+26499\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]+$$

$$\frac{1}{32}\sqrt{\frac{1}{2}\left(-14395+26499\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]$$

Result (type 3, 129 leaves):

$$-17x+\frac{5x^3}{3}-\frac{25x\left(3+x^2\right)}{8\left(3+2x^2+x^4\right)}+$$

$$\frac{\left(-356\,i+127\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{16\sqrt{2-2\,i\sqrt{2}}}+\frac{\left(356\,i+127\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{16\sqrt{2+2\,i\sqrt{2}}}$$

Problem 112: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 225 leaves, 12 steps):

$$5x+\frac{25x\left(1+x^2\right)}{8\left(3+2x^2+x^4\right)}+\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{16}\sqrt{\frac{1}{6}\left(19291+12899\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]+$$

$$\frac{1}{32}\sqrt{\frac{1}{6}\left(-19291+12899\sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 121 leaves):

$$5x+\frac{25\left(x+x^3\right)}{8\left(3+2x^2+x^4\right)}-\frac{\left(-34\,i+111\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{16\sqrt{2-2\,i\sqrt{2}}}-\frac{\left(34\,i+111\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{16\sqrt{2+2\,i\sqrt{2}}}$$

Problem 113: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{\left(3+2x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 224 leaves, 10 steps):

$$\frac{25x\left(1-x^2\right)}{24\left(3+2x^2+x^4\right)}-\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{48}\sqrt{\frac{1}{6}\left(-11567+12897\sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]-$$

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(11567+12897\sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 115 leaves):

$$\frac{1}{48}\left(-\frac{50\,x\,\left(-1+x^2\right)}{3+2\,x^2+x^4}+\frac{\left(95+44\,\mathrm{i}\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1-\mathrm{i}\,\sqrt{2}}}\right]}{\sqrt{1-\mathrm{i}\,\sqrt{2}}}+\frac{\left(95-44\,\mathrm{i}\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1+\mathrm{i}\,\sqrt{2}}}\right]}{\sqrt{1+\mathrm{i}\,\sqrt{2}}}\right)$$

Problem 114: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{x^2\,\left(3+2\,x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 229 leaves, 12 steps):

$$-\frac{4}{9\,x}-\frac{25\,x\,\left(5+x^2\right)}{72\,\left(3+2\,x^2+x^4\right)}+\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{48}\sqrt{\frac{1}{6}\left(-965+699\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}-\sqrt{2\,\left(-1+\sqrt{3}\right)}\;x+x^2\right]+$$

$$\frac{1}{96}\sqrt{\frac{1}{6}\left(965+699\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}+\sqrt{2\,\left(-1+\sqrt{3}\right)}\;x+x^2\right]$$

Result (type 3, 126 leaves):

$$-\frac{4}{9\,x}-\frac{25\,x\,\left(5+x^2\right)}{72\,\left(3+2\,x^2+x^4\right)}-\frac{\left(26\,\mathrm{i}+19\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1-\mathrm{i}\,\sqrt{2}}}\right]}{48\,\sqrt{2-2\,\mathrm{i}\,\sqrt{2}}}-\frac{\left(-26\,\mathrm{i}+19\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1+\mathrm{i}\,\sqrt{2}}}\right]}{48\,\sqrt{2+2\,\mathrm{i}\,\sqrt{2}}}$$

Problem 115: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{x^4\,\left(3+2\,x^2+x^4\right)^2}\,dx$$

Optimal (type 3, 238 leaves, 12 steps):

$$-\frac{4}{27 x^3}+\frac{13}{27 x}+\frac{25 x \left(7+5 x^2\right)}{216 \left(3+2 x^2+x^4\right)}-\frac{1}{432} \sqrt{\frac{1}{6} \left(6073+56673 \sqrt{3}\right)} \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{432} \sqrt{\frac{1}{6} \left(6073+56673 \sqrt{3}\right)} \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{864} \sqrt{\frac{1}{6} \left(-6073+56673 \sqrt{3}\right)} \text{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)} x+x^2\right]-$$

$$\frac{1}{864} \sqrt{\frac{1}{6} \left(-6073+56673 \sqrt{3}\right)} \text{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)} x+x^2\right]$$

Result (type 3, 131 leaves):

$$\frac{1}{864} \left(\frac{4 \left(-96+248 x^2+351 x^4+229 x^6\right)}{x^3 \left(3+2 x^2+x^4\right)}+\frac{2 \left(229+46 i \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{\sqrt{1-i \sqrt{2}}}+\frac{2 \left(229-46 i \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{\sqrt{1+i \sqrt{2}}}\right)$$

Problem 116: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3 x^4+5 x^6}{x^6 \left(3+2 x^2+x^4\right)^2} \, dx$$

Optimal (type 3, 245 leaves, 12 steps):

$$-\frac{4}{45 x^5}+\frac{13}{81 x^3}-\frac{13}{27 x}+\frac{25 x \left(1-7 x^2\right)}{648 \left(3+2 x^2+x^4\right)}+\frac{\sqrt{\frac{1}{6}\left(-1139381+688419 \sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{1296}-$$

$$\frac{\sqrt{\frac{1}{6}\left(-1139381+688419 \sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{1296}-$$

$$\frac{\sqrt{\frac{1}{6}\left(1139381+688419 \sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]}{2592}+$$

$$\frac{\sqrt{\frac{1}{6}\left(1139381+688419 \sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]}{2592}$$

Result (type 3, 140 leaves) :

$$\frac{1}{12960}\left(-\frac{4\left(864-984 x^2+3928 x^4+2475 x^6+2435 x^8\right)}{x^5\left(3+2 x^2+x^4\right)}-\right.$$

$$\left.\frac{10 i\left(-487 i+475 \sqrt{2}\right)\ \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{\sqrt{1-i \sqrt{2}}}+\frac{10 i\left(487 i+475 \sqrt{2}\right)\ \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{\sqrt{1+i \sqrt{2}}}\right)$$

Problem 117: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^{10}\left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^3}\, dx$$

Optimal (type 3, 243 leaves, 13 steps) :

$$58 x-9 x^3+x^5-\frac{25 x \left(15+7 x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}+\frac{x \left(3305+252 x^2\right)}{64 \left(3+2 x^2+x^4\right)}+$$
$$\frac{3}{256} \sqrt{-8595619+7678611 \sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]-$$
$$\frac{3}{256} \sqrt{-8595619+7678611 \sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]+$$
$$\frac{3}{512} \sqrt{8595619+7678611 \sqrt{3}}\ \text{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]-$$
$$\frac{3}{512} \sqrt{8595619+7678611 \sqrt{3}}\ \text{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 156 leaves):

$$58 x-9 x^3+x^5-\frac{25 x \left(15+7 x^2\right)}{16 \left(3+2 x^2+x^4\right)^2}+\frac{x \left(3305+252 x^2\right)}{64 \left(3+2 x^2+x^4\right)}+$$
$$\frac{3 \left(4795\, i+148 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{128 \sqrt{2-2\, i \sqrt{2}}}+\frac{3 \left(-4795\, i+148 \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{128 \sqrt{2+2\, i \sqrt{2}}}$$

Problem 118: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{x^8 \left(4+x^2+3 x^4+5 x^6\right)}{\left(3+2 x^2+x^4\right)^3}\, dx$$

Optimal (type 3, 242 leaves, 13 steps):

$$-27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64(3+2x^2+x^4)}-$$

$$\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\,\text{ArcTan}\Big[\frac{\sqrt{2(-1+\sqrt{3})}-2x}{\sqrt{2(1+\sqrt{3})}}\Big]+$$

$$\frac{21}{256}\sqrt{34271+22721\sqrt{3}}\,\text{ArcTan}\Big[\frac{\sqrt{2(-1+\sqrt{3})}+2x}{\sqrt{2(1+\sqrt{3})}}\Big]-$$

$$\frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\,\text{Log}\Big[\sqrt{3}-\sqrt{2(-1+\sqrt{3})}\,x+x^2\Big]+$$

$$\frac{21}{512}\sqrt{-34271+22721\sqrt{3}}\,\text{Log}\Big[\sqrt{3}+\sqrt{2(-1+\sqrt{3})}\,x+x^2\Big]$$

Result (type 3, 155 leaves) :

$$-27x+\frac{5x^3}{3}+\frac{25x(3+5x^2)}{16(3+2x^2+x^4)^2}-\frac{x(1468+835x^2)}{64(3+2x^2+x^4)}+$$

$$\frac{21(-175i+137\sqrt{2})\,\text{ArcTan}\Big[\frac{x}{\sqrt{1-i\sqrt{2}}}\Big]}{128\sqrt{2-2i\sqrt{2}}}+\frac{21(175i+137\sqrt{2})\,\text{ArcTan}\Big[\frac{x}{\sqrt{1+i\sqrt{2}}}\Big]}{128\sqrt{2+2i\sqrt{2}}}$$

Problem 119: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^6(4+x^2+3x^4+5x^6)}{(3+2x^2+x^4)^3}\,dx$$

Optimal (type 3, 235 leaves, 13 steps) :

$$5x+\frac{25x\left(3-x^2\right)}{16\left(3+2x^2+x^4\right)^2}+\frac{7x\left(11+58x^2\right)}{64\left(3+2x^2+x^4\right)}+$$

$$\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{256}\sqrt{827621+1176531\sqrt{3}}\ \text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\ \text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]+$$

$$\frac{1}{512}\sqrt{-827621+1176531\sqrt{3}}\ \text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

Result (type 3, 138 leaves):

$$\frac{1}{256}\left(\frac{4x\left(3411+5112x^2+4089x^4+1686x^6+320x^8\right)}{\left(3+2x^2+x^4\right)^2}-\frac{i\left(-2644\,i+185\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{\sqrt{1-i\sqrt{2}}}+\frac{i\left(2644\,i+185\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{\sqrt{1+i\sqrt{2}}}\right)$$

Problem 120: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^4\left(4+x^2+3x^4+5x^6\right)}{\left(3+2x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 238 leaves, 11 steps):

$$-\frac{25\,x\,\left(3+x^2\right)}{16\,\left(3+2\,x^2+x^4\right)^2}+\frac{x\,\left(238-59\,x^2\right)}{64\,\left(3+2\,x^2+x^4\right)}-$$

$$\frac{1}{256}\sqrt{3\,\left(-48\,835+32\,827\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{256}\sqrt{3\,\left(-48\,835+32\,827\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{512}\sqrt{3\,\left(48\,835+32\,827\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}-\sqrt{2\,\left(-1+\sqrt{3}\right)}\;x+x^2\right]-$$

$$\frac{1}{512}\sqrt{3\,\left(48\,835+32\,827\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}+\sqrt{2\,\left(-1+\sqrt{3}\right)}\;x+x^2\right]$$

Result (type 3, 129 leaves):

$$\frac{1}{256}\left(\frac{4\,x\,\left(414+199\,x^2+120\,x^4-59\,x^6\right)}{\left(3+2\,x^2+x^4\right)^2}+\frac{3\,\left(174+133\,i\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1-i\,\sqrt{2}}}\right]}{\sqrt{1-i\,\sqrt{2}}}+\frac{3\,\left(174-133\,i\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1+i\,\sqrt{2}}}\right]}{\sqrt{1+i\,\sqrt{2}}}\right)$$

Problem 121: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{x^2\,\left(4+x^2+3\,x^4+5\,x^6\right)}{\left(3+2\,x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 246 leaves, 11 steps):

$$\frac{25\,x\,\left(1+x^2\right)}{16\,\left(3+2\,x^2+x^4\right)^2}-\frac{x\,\left(353+88\,x^2\right)}{192\,\left(3+2\,x^2+x^4\right)}-$$

$$\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\,\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{11}{768}\sqrt{\frac{1}{3}\left(-1825+1089\,\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\,\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\,\left(1+\sqrt{3}\right)}}\right]-$$

$$\frac{11\,\sqrt{\frac{1}{3}\left(1825+1089\,\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\,\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{1536}+$$

$$\frac{11\,\sqrt{\frac{1}{3}\left(1825+1089\,\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\,\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{1536}$$

Result (type 3, 133 leaves) :

$$\frac{1}{768}\left(-\frac{4\,x\,\left(759+670\,x^2+529\,x^4+88\,x^6\right)}{\left(3+2\,x^2+x^4\right)^2}-\right.$$

$$\left.\frac{11\,i\,\left(-16\,i+31\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1-i\,\sqrt{2}}}\right]}{\sqrt{1-i\,\sqrt{2}}}+\frac{11\,i\,\left(16\,i+31\,\sqrt{2}\right)\,\text{ArcTan}\left[\frac{x}{\sqrt{1+i\,\sqrt{2}}}\right]}{\sqrt{1+i\,\sqrt{2}}}\right)$$

Problem 122: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{4+x^2+3\,x^4+5\,x^6}{\left(3+2\,x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 248 leaves, 11 steps) :

$$\frac{25\,x\,(1-x^2)}{48\,(3+2\,x^2+x^4)^2}+\frac{x\,(64+51\,x^2)}{192\,(3+2\,x^2+x^4)}-$$

$$\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{256}\sqrt{\frac{1}{3}\left(-1291+1019\,\sqrt{3}\right)}\;\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2\,x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]+$$

$$\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\;x+x^2\right]-$$

$$\frac{1}{512}\sqrt{\frac{1}{3}\left(1291+1019\,\sqrt{3}\right)}\;\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\;x+x^2\right]$$

Result (type 3, 129 leaves):

$$\frac{1}{768}\left(\frac{4\,x\,(292+181\,x^2+166\,x^4+51\,x^6)}{(3+2\,x^2+x^4)^2}+\frac{3\left(34+21\,i\,\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{\sqrt{1-i\,\sqrt{2}}}+\frac{3\left(34-21\,i\,\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{\sqrt{1+i\,\sqrt{2}}}\right)$$

Problem 123: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3\,x^4+5\,x^6}{x^2\,(3+2\,x^2+x^4)^3}\,dx$$

Optimal (type 3, 253 leaves, 13 steps):

$$-\frac{4}{27x}-\frac{25x\left(5+x^2\right)}{144\left(3+2x^2+x^4\right)^2}-\frac{x\left(325+242x^2\right)}{1728\left(3+2x^2+x^4\right)}+\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}-2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{2304}-\frac{\sqrt{\frac{1}{3}\left(59711+55161\sqrt{3}\right)}\,\text{ArcTan}\left[\frac{\sqrt{2\left(-1+\sqrt{3}\right)}+2x}{\sqrt{2\left(1+\sqrt{3}\right)}}\right]}{2304}-\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}-\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{4608}+\frac{\sqrt{\frac{1}{3}\left(-59711+55161\sqrt{3}\right)}\,\text{Log}\left[\sqrt{3}+\sqrt{2\left(-1+\sqrt{3}\right)}\,x+x^2\right]}{4608}$$

Result (type 3, 140 leaves):

$$\frac{1}{6912}\left(-\frac{12\left(768+1849x^2+1412x^4+611x^6+166x^8\right)}{x\left(3+2x^2+x^4\right)^2}+\frac{3\,i\left(332\,i+7\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1-i\sqrt{2}}}\right]}{\sqrt{1-i\sqrt{2}}}-\frac{3\,i\left(-332\,i+7\sqrt{2}\right)\text{ArcTan}\left[\frac{x}{\sqrt{1+i\sqrt{2}}}\right]}{\sqrt{1+i\sqrt{2}}}\right)$$

Problem 124: Result unnecessarily involves imaginary or complex numbers.

$$\int\frac{4+x^2+3x^4+5x^6}{x^4\left(3+2x^2+x^4\right)^3}\,dx$$

Optimal (type 3, 262 leaves, 13 steps):

$$-\frac{4}{81 x^3}+\frac{7}{27 x}+\frac{25 x \left(7+5 x^2\right)}{432 \left(3+2 x^2+x^4\right)^2}+\frac{x \left(1474+1025 x^2\right)}{5184 \left(3+2 x^2+x^4\right)}-$$

$$\frac{\sqrt{\frac{1}{3} \left(10004741+11240451 \sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}-2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]}{20736}+$$

$$\frac{\sqrt{\frac{1}{3} \left(10004741+11240451 \sqrt{3}\right)}\ \text{ArcTan}\left[\frac{\sqrt{2 \left(-1+\sqrt{3}\right)}+2 x}{\sqrt{2 \left(1+\sqrt{3}\right)}}\right]}{20736}+\frac{1}{41472}$$

$$\sqrt{\frac{1}{3} \left(-10004741+11240451 \sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}-\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]-$$

$$\frac{1}{41472} \sqrt{\frac{1}{3} \left(-10004741+11240451 \sqrt{3}\right)}\ \text{Log}\left[\sqrt{3}+\sqrt{2 \left(-1+\sqrt{3}\right)}\ x+x^2\right]$$

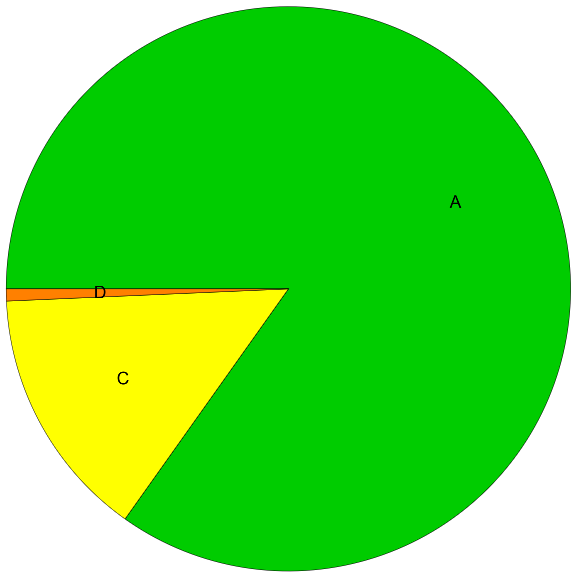
Result (type 3, 139 leaves):

$$\frac{1}{20736} \left(\frac{4 \left(-2304+9024 x^2+20090 x^4+19939 x^6+8644 x^8+2369 x^{10}\right)}{x^3 \left(3+2 x^2+x^4\right)^2}+\right.$$

$$\left.\frac{\left(4738+127 i \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1-i \sqrt{2}}}\right]}{\sqrt{1-i \sqrt{2}}}+\frac{\left(4738-127 i \sqrt{2}\right) \text{ArcTan}\left[\frac{x}{\sqrt{1+i \sqrt{2}}}\right]}{\sqrt{1+i \sqrt{2}}}\right)$$

Summary of Integration Test Results

145 integration problems



A - 123 optimal antiderivatives

B - 0 more than twice size of optimal antiderivatives

C - 21 unnecessarily complex antiderivatives

D - 1 unable to integrate problems

E - 0 integration timeouts